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The (Low-Energy) Physics of the Superstring [and Discussion]

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Phil. Trans. R. Soc. Lond. A 1989 **329**, 373-393
doi: 10.1098/rsta.1989.0084

Phil. Trans. R. Soc. Lond. A **329**, 373–393 (1989)
Printed in Great Britain

The (low-energy) physics of the superstring

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Superstring theories offer the prospect of a finite unified theory of the four fundamental interactions. However, their implications for low-energy physics are difficult to determine because there must be several stages of symmetry breaking leading from the underlying multidimensional supersymmetric string theory, relevant at the scale of string structure (not more than $O(10^{-32}\text{ cm})$), to the four-dimensional non-supersymmetric effective field theory relevant for physics at scales probed in the laboratory (not less than $O(10^{-15}\text{ cm})$). I discuss these stages of compactification, supersymmetry breaking and gauge-symmetry breaking and show how the resulting low-energy theory may closely approximate the observed structure. The implications for the multiplet structure, the masses and the couplings of the observed particle states are considered.

1. Introduction

String theories provide the basis for a finite unification of the strong, the electromagnetic, the weak and the gravitational interations; perhaps they do indeed represent the ultimate 'theory of everything'. However, direct tests for underlying string structure are likely to be very difficult for the string structure is evident only at very short distance scales of order M_P^{-1}, the inverse Planck scales, i.e. $O(10^{-32}\text{ cm})$. The physics at such short distances may have been probed in the very early universe and it is possible some observable relic of this period may exist but, so far as experimental tests in the laboratory are concerned, the 'stringy' aspects of particle states are negligible and tests of an underlying string theory must rely on indirect information such as the spectrum, the masses, the interactions and the couplings of the observed states. These states are essentially massless in Planck units and so laboratory tests concern the zero modes of the string; for this reason this paper is concerned with the physics of these zero modes.

The reason these tests provide only indirect evidence of the string substructure is that between the Planck scale, relevant for string physics, and the scale probed in the laboratory there must be several mass scales associated with further stages of symmetry breaking. There is the compactification scale needed to leave just four flat space-time dimensions. There may be scales associated with gauge-symmetry breaking to reduce the gauge group to that observed at low energies. There is the supersymmetry breaking scale needed to break the four-dimensional supersymmetry and, finally, there is the electroweak breaking scale needed to give mass to the W bosons the quarks and the leptons. These stages of symmetry breakdown serve to obscure the underlying string physics; much of the detailed low-energy structure following from the compactification stage. Although I believe the underlying string dynamics should determine this compactification, at present our analytical techniques do not allow us uniquely to determine the vacuum state of the string. As a result there is a large number of candidate string vacua with a corresponding uncertainty in the string predictions for low-energy physics. Accepting this limitation in our current understanding, it is still possible to make progress by analysing the most promising string vacua to see if their low-energy physics could be consistent with observation. For a given vacuum configuration the spectrum, masses and couplings should be determined so the comparison with experiment is likely to be very challenging. If a viable string model is shown to exist, the multifaceted problem of understanding the spectrum, masses and couplings of the observed states becomes one of trying to understand why a particular vacuum configuration is chosen.

Broadly, this paper falls into two parts. The first is fairly general, surveying the expectations of string theories for low-energy physics. The second part discusses how close specific attempts have come to a realistic low-energy theory. The discussion is restricted to the superstring, string theories with a world-sheet supersymmetry, because only in these theories is there the possibility of a residual supersymmetry in the four-dimensional world, as is needed to provide a solution to the mass hierarchy problem in a theory that does not have composite fields at a low energy-scale.

Briefly, the hierarchy problem (Gildener 1976; Gildener & Weinberg 1976; 't Hooft 1980) is the problem of maintaining a low scale of electroweak breaking in a theory with other much larger scales Λ (in string theories the compactification, M_c, or the Planck scale M_P). In the absence of a space-time supersymmetry there is nothing to prevent the Higgs scalar, Φ, from acquiring a mass term $M^2\Phi^\dagger\Phi$ invariant under all gauge and chiral symmetries. Even if at tree level $M^2 = 0$, radiative corrections induce a mass M at scale μ of $O(\alpha\Lambda^2 \ln(\Lambda^2/\mu^2))$, where $\alpha = g^2/8\pi^2$ following from the one-loop graph with a virtual W exchanged between the Higgs scalar states (where g is the electroweak coupling). Thus the scale Λ^2 of new physics is constrained by $\Lambda^2 \lesssim M_W^2/\alpha$ if the Higgs mass, and hence the electroweak breaking scale, is to be of $O(M_W)$. In the standard model the sceptic might argue that the hierarchy problem is illusory for M^2 must be renormalized. In a string theory, however, one cannot evade the problem for the theory has no divergences and the contribution from the standard model will be cut off at a scale Λ, the compactification or the string scale. Hence the only consistent string theories (if they do not have composite Higgs scalars) must have a low-energy supersymmetry. For example, the W contribution to M^2 is cut off by the wino contribution, leaving a residual contribution as above with $\Lambda^2 = M_{\tilde{W}}^2$, the wino mass related to the supersymmetry breaking scale.

2. String theories

The essential feature of a string theory (Green *et al.* 1987*b*) that distinguishes it from a conventional field theory is that the states in the theory are not pointlike but, at some very short distance scale, have a distribution in a new dimension, σ. Thus string theories start with a world-sheet for the states in the theory, not a world-line. The world-sheet has both a space and a time dimension with coordinates σ and τ. The remaining index, μ, labels the 'target space' that describes the other degrees of freedom of the state, including the normal (four-dimensional) space-time degrees of freedom and further internal degrees of freedom (gauge, etc.). To define a viable theory, the following conditions are imposed.

1. *Modular invariance.* This is equivalent to reparametrization invariance on the world-sheet and requires that physics should not depend on how one parametrizes the underlying world-sheet with coordinates σ and τ.

2. *Conformal invariance.* The action of the string theory is defined in terms of the action on the two-dimensional world-sheet. There is an overall dimensionful coupling (the string tension) but the two-dimensional action itself does not involve dimensionful parameters and is required

to be conformally invariant, i.e. the underlying theory is a two-dimensional conformally invariant field theory.

3. *Absence of anomalies*. By making all states have an extension in the σ dimension, the infinities of conventional field theory associated with point interactions are ameliorated and, it is thought, string field theories give rise to finite theories of the fundamental interactions, including gravity. However, this is only possible if there are no anomalies in the theory, so an additional constraint must be added requiring that the theory be anomaly-free in the gauge and gravitational interactions. The absence of conformal anomalies places a strong constraint on the target space labelled by the μ index ($\mu = 1, \ldots, D$), namely that it should cancel the conformal ghost anomaly, which requires $D = 26$ for the bosonic string and 15 in a supersymmetric theory. As a result $D = 26$ or 10 for the non-supersymmetric and supersymmetric versions of the string respectively in D flat dimensions, because a free bosonic degree of freedom contributes 1 and a fermionic degree of freedom contributes $\frac{1}{2}$ respectively to the conformal anomaly. There is a intimate relation between the absence of anomalies and the modular-invariance constraint discussed above. The local conformal invariance that is achieved once the conformal anomaly is eliminated by the choice of target space corresponds to local reparametrization or modular invariance. It turns out that the requirement of global modular invariance is a necessary and sufficient condition for the elimination of all remaining gravitational and gauge anomalies.

4. *World-sheet supersymmetry*. Although not a necessary requirement for a string theory, I also demand that there be a $N = 1$ supersymmetry in the string theory compactified to $d = 4$ flat space-time dimensions. This allows for a solution of the gauge hierarchy problem so that a natural separation of the scale, M_W, of electroweak breaking from the string scale $(\alpha')^{-1} = O(M_P)$ is possible. As discussed above, the finiteness of the string theory makes the gauge hierarchy problem *more* pressing, and I see no prospect of a viable low-energy theory being built from a non-supersymmetric string theory unless it corresponds to a theory in which light Higgs bosons are composite fields.

3. String compactification

The original realizations of the string conditions listed in §2 identified the D target-space indices with flat space-time, leading to the bosonic string in $D = 26$ dimensions, the fermionic string in $D = 15$ dimensions and the heterotic string with left-moving states described by the bosonic string in $D = 26$ dimensions and the right-moving states described by the fermionic string in $D = 15$ dimensions. As a result these models must undergo a stage of compactification in which all but four of these dimensions are associated with a compact space. The string states may freely propagate in the four flat dimensions, but will be confined to a distance R characterizing the dimension of the compact space in the compact dimension. As a result until distance scales of order R are probed ($R \sim M_c^{-1} \sim 10^{-31}$ cm) the effective theory will appear to have four space-time dimensions. A characteristic feature of a compactified theory is the quantization of the level structure; the imposition of compact boundary conditions causes the levels in split with mass splitting proportional to R^{-1}. As a result a given compactification will make a prediction for the spectrum of states and the comparison of this spectrum for the zero-modes with that observed at low energies, the quarks, the leptons, the gluons, the photons and the W bosons, provides a stringent test of the compactification.

There has been considerable theoretical effort directed towards the analysis of com-

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pactifications consistent with the underlying string dynamics and a large number of possibilities have emerged (J. H. Schwarz, this Symposium).

3.1. *Calabi–Yau compactification*

For the fermionic string the condition the 10-dimensional space should become $M \times K$, where M is four-dimensional Minkowski space and K a six-dimensional compact manifold, together with the condition there should be a residual $N = 1$ supersymmetry led to the conclusion (Candelas *et al.* 1985) that K should have SU(3) holonomy, i.e. a Calabi–Yau space. In the heterotic string the most natural generalization is to identify the spin connection with the gauge connection coming from an SU(3) subgroup of the $E_8 \times E_8'$ gauge group coming from the bosonic sector. As a result the gauge group after compactification is $E_8 \times E_6$. The gauge non-singlet chiral superfields in the theory all originate in the original $E_8 \times E_8'$ gauge bosons. After compactification the E_8' gauge bosons have a harmonic expansion of the form

$$A_\mu^{\tilde{\alpha}}(x, y) = \sum_i^N X_i^\alpha(x)\, Y_{ab}^i(y), \tag{3.1}$$

where we have written the E_8' index $\tilde{\alpha}$ in terms of an E_6 index α and an SU(3) (gauge connection) index b by using the decomposition $E_8 \supset E_6 \times SU(3)$. The index a is an SU(3) space-group index, where we have constructed three complex coordinates a from the six internal coordinates, μ.

Now the E_8 adjoint representation is 248-dimensional, and under $E_6 \times SU(3)$ it decomposes as

$$(248) = (78, 1) + (27, 3) + (\overline{27}, \overline{3}) + (1, 8). \tag{3.2}$$

Consider the representation $(\alpha, b) \equiv (27, 3)$. In this case b is a triplet index and $Y_{iab}(y)$ transforms as a triplet under both space and internal symmetry groups. Because we have identified the spin and the gauge connections, these indices are both SU(3) holonomy indices; Y_{ab} has mixed symmetry under the interchange of a and b, but we may identify it with a differential form by rewriting $Y_{ab}(y) = \epsilon_{acd}\, Y_b^{cd}(y)$. From, for example, equation (3.1) we see the zero modes in four dimensions $X_i(x)$ are associated with zero-modes $Y^i(y)$ in the six-dimensional compact spaces. The number of such fields is given by the number $h_{2,1}$ of related (2, 1) forms $Y \equiv Y_b^{cd}\, dz_A^b\, d\bar{z}_{cA}\, d\bar{z}_d$. Thus one establishes a connection between the Hodge numbers $h_{i,j}$ of the compact manifold and the four-dimensional massless states (Hübsch 1987; see Candelas (1987) for a general review). There are $h_{2,1}$ chiral superfields transforming as the 27 of E_6, $h_{1,1}$ chiral superfields transforming as the $\overline{27}$ of E_6 and a number of gauge single chiral superfields not completely determined by the Hodge numbers but at least the $(h_{2,1} + h_{1,1})$ moduli of the space.

How do these expectations fit with observation characterized by the 'standard model'?

The gauge group is $SU(3) \times SU(2) \times U(1)$, which neatly fits into the $SU(3) \times SU(3) \times SU(3)$ maximal subgroup of E_6. This assignment also fits well for the matter multiplets, for the 27-dimensional representation of E_6 has components transforming as $(1, 3, \overline{3}) \oplus (3, \overline{3}, 1) \oplus (\overline{3}, 1, 3)$ under this maximal subgroup and neatly accommodates a single family of leptons and quarks;

$$(1, 3, \overline{3}) = \begin{bmatrix} H_1^0 & H_2^+ & e^+ \\ H_1^- & H_2^0 & \nu_e \\ e^- & \nu_R & N \end{bmatrix}_R, \quad (3, \overline{3}, 1) = \begin{bmatrix} u \\ d \\ D \end{bmatrix}_L, \quad (\overline{3}, 1, 3) = \begin{bmatrix} u^c \\ d^c \\ D^c \end{bmatrix}_L. \tag{3.3}$$

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Here there is room for the electron, e, its neutrino ν_e and the up and down quarks, u and d. There is also room for Higgs doublets H_1 and H_2. There are additional neutral lepton states ν_R and N and colour triplet states D.

The gauge group E_6 must be broken to give a viable theory and some of the states in equation (3.3) may be expected to be heavy.

However, the picture emerging is quite encouraging, at least the gauge group structure is large enough to accommodate the observed states. (In this context it may be worth recalling that the original extended supergravity models failed because their gauge group structure was just too small.) The family duplication may easily be understood in these models if the Hodge numbers predict more than one excess copy of $27-\overline{27}$ representations (i.e. $h_{2,1}-h_{1,1}$ families). As I discuss in §4 there are only a very small number of Calabi–Yau models with just three families but their existence does support the feasibility of string models as far as their implications for the multiplet structure.

Finally, note that the E_8 sector has not been identified with any of the known states. This is because all known matter states are chiral; they do not belong to real representations and E_8 has only real representations. Attempts to build realistic models from the superstring make use of such a 'hidden' sector to provide a trigger for supersymmetry breaking (Ferrara *et al.* 1983; Derendinger *et al.* 1985; Dine *et al.* 1985*b*; Binétruy & Gaillard 1986; Breit *et al.* 1985*b*; Mangano 1985). It is thought, in analogy with our experience of quantum chromodynamics (QCD), that the E_8 group will be confining and all its states will be massive at the confinement scale. Moreover, the gaugino condensate that seems likely to form in this sector breaks supersymmetry and may act as the trigger for supersymmetry breaking in the 'visible' E_6 sector. Because the two sectors communicate only by gravitational strength, breaking effects are very small in the visible sector (Cremmer *et al.* 1983; Ellis *et al.* 1984*a*, *b*; Coughlan *et al.* 1988; Ross 1988*b*).

3.2. *Four-dimensional string theories*

Subsequent to the discovery of Calabi–Yau compactifications it has been realized that there is a much larger class of possibilities for compactified string models. This may be seen most directly by first observing that the definition of a string given in §2 does not require that all D dimensions be initially interpreted as flat space-time dimensions. Provided the conditions of modular invariance, etc., be realized some of these dimensions may, from the start, be compact or internal. This has led to the study of four string theories (J. H. Schwarz, this Symposium), the set of models satisfying the conditions of §2 with just four flat space-time dimensions.

3.2.1. *Orbifold models*

The simplest compactified models use flat or toroidal compactification to describe the internal dimensions. Thus, for example, six of the fermionic string dimensions may be identified with the real six torus $(T_R)^6$ defined by a $[SU(3)]^3$ lattice. This has an $N = 4$ supersymmetry in four dimensions, too large for a realistic theory, so to reduce this to an $N = 1$ supersymmetry the torus is modded out by a discrete point group P to form and 'orbifold' that breaks the $N = 4$ supersymmetry to an $N = 1$ supersymmetry introducing curvature at a discrete set of points (this is the discrete analogue of the SU(3) holonomy group met in Calabi–Yau compactification where, however, the curvature is distributed throughout the manifold). For the simple example the point group may be chosen as Z_3, the discrete rotations of the three SU(3) lattices simultaneously by 120°. The orbifold is known as the Z_3 orbifold (Dixon *et al.*

1985, 1986; Ibañez *et al.* 1987*a*, *b*; Chamseddine & Derendinger 1987). The action of P leaves $3\times3\times3=27$ fixed point, so the structure thus defined is an orbifold, not a manifold. In the orbifold there are two types of closed strings: 'untwisted' and 'twisted'. The untwisted string is one closed on the torus. The twisted strings rely on an action of the point group to close. The simplest heterotic versions of orbifold models use the same orbifold construction for left- and right-movers,

$$K = T^6_R/P \otimes T^6_L/P \otimes T^{16}_L/G, \tag{3.4}$$

where there may also be isometries G of the additional gauge degrees of freedom in the bosonic sector reducing the gauge group after compactification. These orbifold models have a discrete holonomy group, a subgroup of the SU(3) holonomy group used in Calabi–Yau compactification. As a result the gauge group structure (with trivial gauge isometries) is $E_8\times E_6\times G_0$ where G_0 is a subgroup of SU(3). For the case of abelian orbifolds, $G_0 = SU(3)$, $SU(2)\times U(1)$ or $U(1)^2$.

Orbifold models have been extensively studied, and it has been shown how to go beyond the left–right symmetry assumption that went into the Calabi–Yau construction to form asymmetric orbifold compactifications,

$$K = T^6_R/P_R \otimes T^6_L/P_L \otimes T^{16}_L/G. \tag{3.5}$$

It also proves possible to use the full 22 $(=26-4)$ internal dimensions of the bosonic string to provide gauge degrees of freedom leading to the possibility of a rank 22 gauge group (Narain 1986; Narain *et al.* 1986, 1987). Of course this is much too large to describe the gauge symmetries observed at low energies, and considerable effort has gone into finding 'realistic' orbifold models (Ibañez *et al.* 1987*a–c*, 1988; Bailin *et al.* 1987; Casas *et al.* 1987; Casas & Muñoz 1988). The best candidate models now have three generations of quarks and leptons and with non-trivial gauge isometries and a gauge group $SU(3)\times SU(2)\times U(1)^n$, where $n>1$. Because the curvature is only at isolated points, the advantages of toroidal compactification are kept and all the properties of the model may be computed (Cvetic 1987).

3.2.2. *Fermionic formulation*

The formulation of string theories in spaces with isolated points of curvature has also been extensively explored using the 'fermionic' formulation in which the internal degrees of freedom are described by fermionic coordinates (Kawai *et al.* 1986; Lerche *et al.* 1987; Müller & Witten 1986; Antoniadis *et al.* 1987*a*; Schellekens 1987). Although partly overlapping the bosonic orbifolds formulation it also gives rise to new compactification not easily related to the orbifold construction. Once again it has been shown possible to build three-generation models, as I discuss in the next section.

3.2.3. $N=2$ *discrete series*

A recent development seeks to construct the string theory in four dimensions by using tensor products of $N=2$ unitary discrete series chosen to cancel the conformal anomaly (Gepner 1988). This technique provides a generalization of the orbifold and fermionic methods allowing for the construction of supersymmetric conformal field theories with curvature.

Because the $N=2$ discrete series is completely solvable they retain the advantages of the other formulations in that the partition function is completely known at string tree level. Remarkably, these models generate some complete intersection Calabi–Yau (CICY) theories allowing for the complete computation in these theories of the spectrum and couplings.

The $N = 2$ minimal models are labelled by a positive integer, k, and the kth models has a construction to the central charge (i.e. the contribution to the conformal anomaly) given by $c = 3k/(k+2)$.

4. Low-energy implications of string compactifications

In this section I briefly review the characteristics to be expected for the low-energy physics following from string compactifications. Because of the large number of candidate string vacua it is not possible at present to make definitive statements, but certain features seem to be common to a wide class of realistic compactification schemes; in the next section I illustrate how these generic features are realized in specific models.

4.1. *Vacuum structure*

In addition to the problem of the large number of disconnected string compactifications within a particular scheme there may also be several undetermined parameters. The first set of these correspond to the moduli, which determine the shape and size of the manifold. In Calabi–Yau models there are $(h_{2,1}+h_{1,1})$ such moduli that correspond to chiral supermultiplets whose scalar components have undetermined vacuum expectation values (VEVs) because their potential vanishes identically. However, the symmetries of the model may be such that the potential of many other fields also vanishes identically, or to a high degree of accuracy, giving rise to further 'flat' directions in the potential. It is along such flat directions that gauge non-singlet fields may acquire large VEVs reducing the gauge symmetry.

The determination of these VEVs occurs only once the vacuum degeneracy along the flat direction is lifted. This happens once the low-energy $N = 1$ supersymmetry is broken as is expected to happen via the formation of condensates in the hidden sector. In supergravity theories such a gaugino condensate breaks supersymmetry (Ferrara *et al.* 1983; Derendinger *et al.* 1985; Dine *et al.* 1985*b*; Binétruy & Gaillard 1986; Breit *et al.* 1985*b*; Mangano 1985) giving rise to a gravitino mass and supersymmetry breaking in the hidden sector. These effects are communicated by gravitational-strength couplings to the visible sector giving masses to the scalar components of chiral superfields and the fermion component of gauge supermultiplets (Cremmer *et al.* 1983; Ellis *et al.* 1984*a*, *b*; Coughlan *et al.* 1988; Ross 1988*b*). Because of large radiative corrections involving the light states in the theory some of the squared scalar masses are driven negative triggering spontaneous symmetry breakdown as the scalar acquires a VEV at the minimum of the potential (Inoue *et al.* 1982, 1984; Yamagishi 1983; Alvarez-Gaume *et al.* 1983; Ibañez *et al.* 1985; Gato *et al.* 1985). Detailed studies (Ross 1988) of these effects show that they do indeed fix the moduli, often at values enhancing the residual discrete symmetries of the model. It seems likely, too, that gauge symmetries will be broken at scales close to the compactification scale whenever there are flat directions involving gauge-non-singlet fields. Thus a large part of the vacuum degeneracy is lifted via supersymmetry breaking effects, going some way towards answering the big question of why a particular vacuum configuration is chosen. This applies to the set of string vacua continuously connected via change of moduli or other VEVs, for the small (on the compactification scale) supersymmetry breaking effects may still cause these parameters to change to reduce the vacuum energy. However, different string vacua not continuously connected will not be driven to a unique ground state by these small supersymmetry breaking effects because the potential energy barrier between these vacua is of the order of the compactification scale. The question of how the choice between such vacua is made is a question of how the universe evolved from early times at which it had a temperature of the order of the compactification scale. Thermal effects do not respect supersymmetry because of the different statistics obeyed by fermions and bosons, so we may expect these effects to split the string vacua degeneracy and consequently a study of string vacua at high temperature may show how the true vacuum state is chosen from the set separated by large potential barriers.

4.2. *Gauge and discrete symmetries*

As has already been stressed the gauge symmetry is determined by the compactification and the only definite constraint is that it should have rank 22 or less with a supersymmetry $N = 4$ or less. Specific vacua have much smaller symmetry groups and here I concentrate on models that have a gauge symmetry given by E_6 or smaller together with a hidden-sector gauge group that plays no role in the low-energy structure apart from providing a trigger for supersymmetry breaking needed to solve the gauge hierarchy problem. Although it seems attractive to have a compactification scheme that has just the standard $SU(3) \times SU(2) \times U(1)$ gauge group this is by no means essential for there may be stages of gauge symmetry breaking by the usual Higgs mechanism after compactification in which scalar fields acquire vacuum expectation values. Whether this happens is determined by the form of the scalar potential following from the compactification, and the occurrence of flat directions involving gauge non-singlet fields. The existence of such flat directions seems to be a generic feature in string compactifications and so the general expectation for the low-energy gauge group is that it should be much smaller than the maximal rank 44 gauge group. As is seen in specific models it is easy to reduce the gauge symmetry to just that of the standard model at a high scale, the question becomes one of why there is *any* residual gauge symmetry. The answers to this relies on whether the flat directions are 'exhausted' before all gauge symmetries are broken; specific models may give rise to additional light gauge bosons (Dine *et al.* 1985*a*; Breit *et al.* 1985*a*; del Aguila *et al.* 1986).

In part, the existence of flat directions is due to additional discrete symmetries present in the theory after compactification (Witten 1986; Greene *et al.* 1986). Their origin lies in the discrete symmetries of the compactification manifold or in the underlying two-dimensional conformal field theory and their trace in the low-energy theory may provide a good signal of the underlying physics. In specific models they may play an important role in suppressing baryon-number-violating decay processes and limiting the form of the light quark- and lepton-mass matrices (Greene *et al.* 1986, 1987).

4.3. *Matter content*

The standard model has matter content that comes in three copies or families of a distinctive multiplet pattern of quarks and leptons. The topological structure of the compactification scheme determines the matter content and thus may provide an explanation for this family duplication and the multiplet structure within a family. There may also be left light, new states not needed for the standard model. For example, each family in E_6 models is 27 dimensional and has an additional charge $Q = -\frac{2}{3}$ coloured state, a right-handed neutrino state, plus two charged lepton and five neutral lepton helicity states. These states may receive mass on the spontaneous breaking of the gauge symmetry, but there are often left light some residue of such new states.

4.4. *Couplings, masses and mixing angles*

Perhaps the most exciting prospect for testing string theories lies in the fact that the couplings of the model are, in principle, fixed on compactification. In practice the analytical techniques available may not allow all this information to be determined. In Calabi–Yau compactifications the metric is not known and so only limited information on couplings based on topology has been obtained. However, in constructions based on free fields the couplings are determined and this may sometimes be used to determine the couplings in other models. For example, orbifold calculations may be extended to the associated Calabi–Yau models related via blow-up of the orbifold singularities. The recent models based on tensor products of minimal $N = 2$ theories may, more directly, determine the couplings in Calabi–Yau models at particular points in moduli space (Distler & Greene 1988; Gepner 1987). These predictions do not rely on discrete topological invariants and so, unlike the predictions for spectra, they may be sensitive to corrections such as non-perturbative effects, higher loop corrections, etc. The question of the stability of coupling predictions has received much attention and it has been shown it is possible to obtain reliable predictions for some of the couplings in the theory responsible for mass generation in the standard model (Distler & Greene 1988; Lutken & Ross 1988). In particular, using the world-sheet conformal symmetry (Dixon 1987) one may show that the $(27)^3$ couplings do not receive non-perturbative world-sheet instanton corrections and so their calculation in the conformal field theory limit that corresponds to a particular compactification radius may be used for the Calabi–Yau theory at arbitrary radius. The couplings, $(\overline{27})^3$, do, however, receive large non-perturbative corrections and so the conformal field theory estimate does not apply at arbitrary radius. However, these couplings are determined via topological techniques in the field theory limit and the non-perturbative corrections about this limit are likely to be small and in some cases can be calculated. (Similar considerations may be applied to the other trilinear couplings of the model.) Beyond string tree level few calculations have been made but, in theories with a residual supersymmetry, the non-renormalization theorems ensure that these corrections to couplings are small.

Being an effective field theory below the compactification scale, there are also non-renormalizable couplings to be considered. It has been shown that these only occur in non-perturbative order (Cvetic 1987; Burgess *et al.* 1986) and so, if the compactification scale is below the Planck scale (as is expected if the hierarchy problem is to be solved), these terms will be small. They may, however, still be important in theories with large-scale VEVs developing along flat directions; in particular, neutrino masses are quite sensitive to non-renormalizable terms (Greene *et al.* 1987; Nandi & Sarkar 1986). At present there are few explicit calculations of non-renormalizable couplings, but the possible terms are strongly constrained by the gauge and discrete symmetries of the compactification scheme. Given a knowledge of the couplings of the theory it is, in principle, possible to determine all of the properties of the low-energy theory. In particular, after electroweak breaking the cubic couplings will determine the quark and lepton mass matrices and hence their masses and mixing angles. One major difficulty in implementing this goal is the uncertain origin of supersymmetry breaking which is important in triggering various stages of gauge symmetry breaking, including the electroweak breaking stage. As I shall discuss, however, with a plausible model for this supersymmetry breaking quite realistic mass matrices may emerge in specific models.

5. Models for low-energy physics

The ideal scenario in choosing a model for low-energy physics is to examine the possible vacuum configurations of the string and ask which one will be chosen in the evolution of the universe. Some progress has been made in this direction, but certainly not sufficient to choose between the many candidate vacua. It therefore seems reasonable, at this stage, to invoke phenomenological criteria when selecting a vacuum solution. The most obvious constraint is that this solution should reproduce the correct multiplet structure at low energies, and this leads to the consideration of three-generation models. It should also be supersymmetric below the compactification scale if we are to have a hope of solving the hierarchy problem. Processes violating baryon-number and lepton-number in such models can proceed very quickly and there should be a mechanism for suppressing these processes.

In this section I discuss four promising three-generation schemes to illustrate just how far one can get towards a viable theory from a definite vacuum solution of the string equations of motion.

5.1. *Calabi–Yau models*

Of the known Calabi–Yau models, only one distinct variety (of the CICY type) is known which has just three generations (Aspinwall *et al.* 1987; Candelas *et al.* 1988; Green *et al.* 1987*a*). Three different manifold constructions of the model are known based on $CP_2 \times CP_2$, $CP_3 \times CP_3$ (Yau 1985) and $CP_3 \times CP_2$ (Schimmrigk 1987), respectively. The simplest, which does not require any blowing-up of singularities, starts with $R_0 = CP_3 \times CP_3$. We call its complex coordinates $x_i, y_i, i = 0, 1, 2, 3$. The three-generation model is based on the quotient manifold $R = R_0/G$, where G is the freely acting Z_3 discrete group defined by

$$\left.\begin{aligned} g&:(x_0\, x_1\, x_2\, x_3) \rightarrow (x_0\, \alpha^2 x_1, \alpha x_2, \alpha x_3),\\ g&:(y_0\, y_1\, y_2\, y_3) \rightarrow (y_0, \alpha y_1, \alpha^2 y_2, \alpha^2 y_3).\end{aligned}\right\} \tag{5.1}$$

The six complex coordinates are reduced to the three needed for a space of SU(3) holonomy by the constraints,

$$\left.\begin{aligned} &\textstyle\sum x_2^3 + a_1 x_0 x_1 x_2 + a_2 x_0 x_1 x_3 = 0,\\ &\textstyle\sum y_2^3 + b_1 y_0 y_1 y_2 + b_2 y_0 y_1 y_3 = 0,\\ &x_0 y_0 + c_1 x_1 y_1 + c_2 x_2 y_2 + c_3 x_3 y_3 + c_4 x_2 y_3 + c_5 x_3 y_2 = 0.\end{aligned}\right\} \tag{5.2}$$

The model has vanishing first Chern class, an Euler characteristic of -6 corresponding to three generations and Hodge numbers (Greene *et al.* 1986, $h_{2,1} = 9$ and $h_{1,1} = 6$ corresponding to nine copies of 27 and six of $\overline{27}$. The model has a complex structure determined by the $h_{2,1} = 9$ coefficients of equation (4.2) that should be interpreted as VEVs of the scalar components of the nine chiral supermultiplets the $h_{2,1}$ moduli (Ross 1988*b*). For a special choice of these parameters $a_i = b_i = 0$ and only c_{ii} non-vanishing with $c_{00} = c_{11} = 1$, $c_{22} = c_{33} = c$ the manifold has a large set of discrete symmetries (Greene *et al.* 1986) that imply the existence of several nearly flat directions among the gauge non-singlet fields. It can be shown that after supersymmetry breaking this choice corresponds to a local minimum of the potential describing the moduli (Ross 1988*b*) showing how a particular complex structure may be chosen.

The phenomenology of this model (Greene *et al.* 1986, 1987) has been examined assuming this symmetric point and also assuming a non-trivial embedding of the Z_3 equation (5.1) in the gauge group. (There are only three possible such embeddings so this assumption does not amount to 'fine-tuning' of the vacuum structure.) With this embedding the gauge group is broken to $[SU(3)]^3$ and the multiplet structure consists of nine generations plus six antigenerations of leptons in the $(1,3,\overline{3})$ representation plus seven generations and four antigenerations of quarks in both the $(3,\overline{3},1)$ and $(\overline{3},1,3)$ representations. The flat directions allow for just two stages of large gauge symmetry breaking (i.e. very much greater than M_W) reducing the gauge group to $SU(3)\times SU(2)\times U(1)$. This is because after these two stages all $\overline{27}$ components have acquired large masses so there are no further D flat directions along which large VEVs may develop. Because the trilinear couplings $(\overline{27})^3$ are completely known (Distler *et al.* 1987) in this model it is possible to compute the radiative corrections and verify that supersymmetry breaking effects will trigger the gauge symmetry breaking along the flat directions (Yamagishi 1983; Alvarez-Gaume *et al.* 1983; Ibañez *et al.* 1985; Gato *et al.* 1985). The states left light down to the electroweak breaking scale consist of just three generations of quarks and leptons, together with their superpartners, one pair of Higgs doublets (H_1^0, H_1^-), (H_2^+, H_2^0), the minimum necessary to generate electroweak breaking, together with their superpartners. The only additional states are those in three neutral and one charged lepton superfield whose masses are of order 1 TeV. The new $Q=-\frac{1}{3}$ colour states in triplet the $(3,\overline{3},1)$ and $(\overline{3},1,3)$ representations all acquire the large gauge breaking mass.

5.2. *Orbifold models*

Much work has gone into the construction of orbifold models both with (2,0) and (2,2) world-sheet supersymmetry and realistic versions have now been developed. These models rely on flat directions to reduce the rank of the gauge group at a high scale. A typical version of such models (Ibañez *et al.* 1987*a–c*, 1988) starts with a Z_3 orbifold with non-trivial embedding of the point group in the gauge group leading to a gauge group after compactification,

$$SU(3)\times SU(2)\times U(1)^8\times SO(10), \tag{5.3}$$

and a spectrum consisting of three generations plus numerous exotic particles. The symmetries are found to require the existence of numerous flat directions along which the VEVs of various fields in the theory are undetermined. A selection criterion is imposed by hand that requires that VEVs should only develop along flat directions which leave the $U(1)_Y$ of the standard model unbroken. If large VEVs along these allowed directions are assumed, then it is found that the unwanted colour triplets in the model, which may mediate proton decay, became massive along with many others of the extra particles. In this respect the model is similar to the Calabi–Yau model discussed above.

The origin of the large VEVs in these models follows from the fact that they always have one anomalous $U(1)_X$, which gives a non-vanishing contribution $(g/192\pi^2)\,\mathrm{Tr}\,[X]$ to the associated D term. Then the complete D term is given by

$$D^X = \Sigma_a q_a^X |\Sigma_a|^2 + \frac{g}{192\pi^2}\mathrm{Tr}\,[X], \tag{5.4}$$

where Σ_a are the scalar components of chiral superfields in the theory and q_a^X is their X charge. Minimization of the vacuum energy will require that some of the fields Σ_a develop VEVs of the order of the compactification scale, the minimum of the potential corresponding to VEVs developing along the 'flat' directions. Note, however, that there are further flat directions in general whose VEVs are not determined by the anomalous F term, as they happen to be invariant under X.

To proceed further, it is necessary to select one among the many candidate flat directions, assigning arbitrary VEVs. When they are not determined by the condition, D^X vanishes. Along a phenomenologically promising choice chosen by 'hand' with gauge group $SU(3) \times SU(2) \times U(1)_Y$, it is found that all exotic quarks and leptons get a large mass, leaving just the quark and lepton states needed for the supersymmetric standard model plus three pairs of doublet and antidoublet Higgses and some massless hypercharge singlets. In addition, there is an $SO(10) \times U(1)$ hidden sector. Again, the resultant structure is very similar to the Calabi–Yau model discussed above.

The phenomenological implications of the models have been broadly studied (Ibañez *et al.* 1987*a–c*, 1988; Bailin *et al.* 1987; Casas *et al.* 1987; Casas & Muñoz 1988). It is found that proton decay is inhibited because of the absence of $qq\bar{q}l$ couplings (see §6.1). There may be an additional light $U(1)'$ gauge boson coupled only to leptons, its mass is determined by VEVs not fixed by equation (5.4). If this is present, it guarantees the absence of the $qq\bar{q}l$ couplings; if not, the absence of $qq\bar{q}l$ follows only if the particular flat direction considered is chosen. Quark and lepton masses arise after electroweak breaking provided the relevant Yukawa couplings to the Higgs doublets are present. At tree level, the symmetries of the model allow only one set of Yukawa couplings, so that only the heaviest quarks and leptons (t, b, c, τ) can acquire tree-level masses. It may be that the remaining states acquire masses through loop corrections, although this has not been studied and it is difficult to see how sufficiently large masses will come from such a mechanism.

It is interesting that, again, a multiplet structure very close to the supersymmetric standard model is obtained in a specific model, but once more the outstanding question is why the particular flat direction is chosen. The generic expectation seems to be that the large rank of the gauge group normally found in orbifolds will be reduced due to the flat directions, although it is of course, in principle, possible that additional light $U(1)$ gauge bosons persist in low-energy scales.

5.3. *Flipped* SU(5) (*Antoniadis et al.* 1987*b*)

Early attempts to construct viable models based on the heterotic superstring used non-trivial gauge group embedding of the quotient group to reduce the gauge group from $E_8 \times E_6$. It was quickly realized that the maximal size of the resultant gauge group is severely constrained for, in general, the subsequent breaking at a high scale of the gauge symmetry by the usual Higgs mechanism will leave an SU(5) group intact and the SU(5) gauge interactions then generate nucleon decay at an unacceptable rate. The reason for this is obvious with the conventional E_6 assignment of particles, for the only states which are singlets under the standard model gauge group, $SU(3) \times SU(2) \times U(1)$ (these are the only states which can receive large (very much greater than M_W) VEVs) are ν_R and N, and these leave an SU(5) subgroup of E_6 intact. The trouble is that the breaking of SU(5) in this case requires an adjoint representation of Higgs scalars, but in superstring models the chiral supermultiplets do not come in adjoint representations.

There is an exception to this rule, the so-called 'flipped' $SU(5) \times U(1)$, in which the assignments of the u_L and d_L quarks to SU(5) representations are interchanged (Barr 1982; Derendinger *et al.* 1984). This is not possible in usual SU(5) because the charge operator is a

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generator of SU(5), so the charge should be traceless over a representation, but in SU(5) × U(1) there is a U(1) component of the charge allowing for this change. Moreover, in the breaking of E_6 it is not unreasonable that such additional U(1) factors should appear. With this assignment, it is possible to break SU(5) at a high scale down to the standard model group by using just 5, $\bar{5}$, 1 and 10 representations of SU(5), and these representations may be present in superstring compactification.

There is a potential problem for such a grand unified scheme that did not arise in the problems so far discussed, namely the doublet–triplet splitting problem. This arises because a grand unified group such as SU(5) requires the existence of Higgs colour triplets as degenerate partners of the Higgs doublets. These, and their fermion superpartners, can mediate proton decay and should be made very massive, of the order of 10^{16} GeV, while keeping their doublet partners, needed for electroweak breaking, light, of $O(M_W)$. The superstring theories so far discussed neatly avoid this problem, for their initial gauge groups are *smaller* than SU(5) and so imply no symmetry relation between SU(2) doublets and SU(3) triplets. This represents a significant improvement over the general grand unified theory (GUT).

In the case of a flipped SU(5), however, the problem is present, but happily is solved by the missing partner mechanism (Masiero *et al.* 1982; Grinstein 1982; Kounnas *et al.* 1983; Campbell *et al.* 1987), in which the couplings of the 5 and $\bar{5}$ of Higgses contain just the required triplet mass term but no doublet mass term. This is not in conflict with the underlying SU(5) symmetry, for the symmetry is broken by the large VEV of the ν_R component of the 10, and only the colour triplets couple to the ν_R component.

Flipped SU(5) has been discussed in the context of manifold compactification (Antoniadis *et al.* 1988*a*), but no explicit realization is known. Recently, a four-dimensional string construction with flipped SU(5) has been developed (Antoniadis *et al.* 1988*b*) that predicts the multiplet structure as well as the gauge group. The most promising version has an 'observable' GUT group SU(5) × U(1) × U(1)^3 with three generations of matter field with SU(5) × U(1) transform properties:

$$F_i = (10/\tfrac{1}{2}), \quad \bar{f}_i = (5, -\tfrac{3}{2}), \quad T_1^c = (1, \tfrac{5}{2}); \quad i = 1, 2, 3.$$

In addition there are the 10-dimensional Higgses needed to break SU(5) × U(1) down to the standard group,

$$H_{1,2} = (10, \tfrac{1}{2}), \quad \bar{H}_{1,2} = (10, -\tfrac{1}{2}),$$

and the five-dimensional Higgses needed to break the standard model,

$$h_{1,2,3} = (5, -1), \quad \bar{h}_{1,2,3} = (\bar{5}, 1).$$

There are also three pairs of singlets and their complex conjugates:

$$\varphi_{12} = (1, 0), \quad \varphi_{23} = (1, 0), \quad \varphi_{13} = (1, 0).$$

The model requires a stage of large symmetry breaking, which occurs along a flat direction. Unlike the previous models there are not a large number of flat directions to consider, so it is easier to establish the breaking pattern. There is, in addition to the visible sector multiplets, an '*à la carte*' hidden sector that may be responsible for supersymmetry breaking, although its properties and flat directions have not been explored to date.

After GUT breaking, one is left with basically the supersymmetric standard model, possibly with the addition of extra U(1) gauge bosons and Higgs doublets and singlets; the details have not been completely determined. The GUT-breaking scale is large, $M_G \approx 10^{16\pm1}$ GeV, the dominant nucleon decay is via dimension-six operators (see §6.1) and is expected to be very slow ($10^{33}y \ll 10^{57}y$).

Fermion masses are determined by undefined Yukawa couplings, but some general properties of the masses are known (Antoniadis *et al.* 1988*b*; Leontaris & Nanopoulos 1988; Leontaris 1988). There is an SO(10) relation $m_b = m_t$ at the Planck scale which, after QCD renormalization, gives m_b of O(6 GeV). The top quark mass is found to be of O(70 GeV) if electroweak breaking via radiative corrections is to be triggered. The structure of the superpotential consistent with the known symmetries of the model can give an acceptable pattern of fermion masses.

5.4. *Construction by using $N = 2$ superconformal theories*

Recently Gepner (Gepner 1987) has constructed a version of the three-generation Calabi–Yau theory by using the tensor product of one $k = 1$ theory and three $k = 16$ theories. The gauge group before gauge symmetry breaking is $E_8 \times E_6 \times U(1)^3$ and the multiplet structure is in agreement with that found in the Calabi–Yau version. In addition, he is able to determine that there are 62 gauge singlet chiral superfields, three of which are needed to give mass to the three additional gauge bosons as one moves away from the exceptional point in moduli space. The symmetries of the model do not, however, correspond to the model constructed above, but rather to the model constructed by Schimmenck (1987) using the $CP_2 \times CP_3$ manifold.

The advantage of this version of the model is that all the trilinear couplings have been calculated, and extensive information about the E_6 gauge-singlet structure and the symmetries has been derived (Sotkou & Stanishkov 1988; Greene *et al.* 1988).

In constructing this version of the model it is necessary to mod out by a $Z_3 \times Z_3$ group, the first Z_3 being freely acting and the second not (in the manifold construction the resultant singularities must be resolved). If I also assume a non-trivial embedding of the first Z_3 in the gauge group we get a gauge group $SU(3)^3 \times U(1)^3$ and the same multiplet structure given as I discussed in §5.1. However, if the second Z_3 is embedded in the gauge group, although the same gauge group obtains, a different multiplet structure results for the quarks for only three generations survive in the $(3, \bar{3}, 1)$ and $(\bar{3}, 1, 3)$ representations. So far only this latter case has been analysed (Lutken & Ross 1989) using the new information on symmetries and couplings coming from the conformal field theory construction. In this case it is straightforward to determine whether there are any stages of symmetry breaking subsequent to compactifications by studying the renormalization group equations. Again it is found there will be gauge breaking and there is a local minimum in which the gauge group is reduced to $SU(3) \times SU(2) \times U(1)$.

6. Phenomenology of the low-energy model

Having discussed the expectations for the low-energy multiplet structure in general and in particular, I turn now to a résumé of the low-energy phenomena to be expected from string models in four flat space-time dimensions.

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6.1. *Baryon-number and lepton-number violation*

An essential ingredient for a viable low-energy model is that violation of the baryon and lepton numbers should be within experimental limits. However, all supersymmetric models suffer from possible baryon-number violation through dimension-four operators of the form

$$[\mathrm{lh_1, ll e^c, q l d^c, u^c d^c d^c}]_F, \tag{6.1}$$

where the superfields l and q refer to left-handed (LH) doublets of leptons and quarks under $SU(2)_L$; e^c, u^c and d^c refer to LH singlet antileptons and antiquarks, respectively; and h_1, h_2 are LH Higgs superfields. The presence of these operators leads to processes violating lepton and baryon number unsuppressed by an inverse powers of mass greater than the supersymmetry-breaking scale (O(1 TeV)), and hence some, or all, of these operators should be absent at tree level (Weinberg 1982; Sakai & Yanagida 1982). To achieve that it is normally assumed that there is a discrete symmetry of the superpotential, called matter parity, under which quark and lepton superfields change sign, while all other superfields remain the same. This forbids operators of the type given in equation (5.1), while allowing the operators needed for mass generation, namely,

$$[\mathrm{lh_2 e^c, qh_2 d^c, qh_1 u^c, h_1 h_2}]_F, \tag{6.2}$$

where h_1 is the LH Higgs supermultiplet of the standard model needed to give u-quarks a mass, and h_2 is the one needed for d-quarks and leptons. The last term in (6.2) is needed to allow for an acceptable pattern of VEVs for h_1 and h_2.

Although original expectations were that it would be difficult to find string models that achieved the necessary suppression that has not proved to be the case because of the additional gauge and discrete symmetries predicted by the compactification. In the Calabi–Yau model (Greene *et al.* 1986, 1987) and also in the $N = 2$ superconformal version (Lutken & Ross 1989) R parity emerges naturally. For the Calabi–Yau model of §5.1 there is a Z_2 group with the action

$$x_2 \leftrightarrow x_3; \quad y_2 \leftrightarrow y_3. \tag{6.3}$$

This clearly leaves the defining equation (5.2) invariant for the maximally symmetric manifold. For the $N = 2$ theory of §5.4 there is a Z_2 group under which twisted modes are odd and untwisted even. Although both of these symmetries are spontaneously broken after compactification they combine with a Z_2 subgroup of the residual gauge group to leave an unbroken R parity as required. In the orbifold and fermionic string model it turns out that gauge symmetries prevent the appearance of the operators of (5.1) (Ibañez *et al.* 1987*a–c*, 1988; Bailin *et al.* 1987; Casas *et al.* 1987; Casas & Muñoz 1988; Antoniadis *et al.* 1988*b*).

Although the models have eliminated dimension-four baryon-number violation, at dimension five and beyond there *are* baryon- and lepton-number violating terms involving light states.

Let us first consider baryon-number violation mediated by the D-quarks of the model. The dimension-five operator contribution gives a proton decay amplitude proportional to M_D^{-1}. The calculation of the proton decay rate following from this contribution involves considerable uncertainty due to the unknown Yukawa couplings and supersymmetry-breaking effects, but it has been plausibly argued (Weinberg 1982; Greene *et al.* 1988) that these graphs will lead to unacceptable rates for proton decay unless the massive colour triplet states mediating the decays satisfy the bounds $M_D > 10^{16}$ GeV. Studies of the potential governing the large breaking scale suggest that breaking scales large enough to satisfy this bound are quite reasonable.

In the flipped SU(5) model the dimension-five terms are absent so proton decay proceeds only via dimension-six operators with amplitude proportional to M_{G}^2 and likely to be invisible.

Lepton-number-violating processes for the models turn out to be well within current experimental limits for breaking scales consistent with baryon-number-violation limits.

6.2. *Neutrino masses*

The neutrinos belong to the lepton doublets and can acquire Dirac masses by coupling with the singlet-lepton components. These mass terms arise only after electroweak breaking from terms allowed by the discrete symmetries, which may be expected to be of the same order as the charged-lepton masses m_1. In addition, we must consider the effect of the large Majorana mass terms for the singlet-lepton components. The usual diagonalization of the neutrino mass matrix leaves light neutrinos with Majorana mass of the order of m_1^2/m^*, where m^* is the mass of the singlet component with Dirac coupling to the neutrino.

In models with a large-scale, M_I, of gauge symmetry breaking the expectation is that Majorana mass will be large, coming via dimension-four non-renormalizable terms in the superpotential and giving m^* of the order of M_I^2/M_c. This gives (Greene *et al.* 1986, 1987; Nandi & Sarkar 1986) neutrino masses of the order of $m_t^2\, m_c/M_1^2$. For $m_1 = 10^{16}$ GeV, this is of order 10^{-11}, 10^{-7} and 10^{-5} eV for the e, μ or τ, respectively, well within current bounds on Majorana neutrino masses.

6.3. *The weak mixing angle*

After the compactification the gauge couplings remain related as if embedded in a GUT. This means that the weak mixing angle is fixed, and for E_6-based models the prediction is $\sin^2\theta_w = \frac{3}{8}$. However, this value gets renormalized as one continues the prediction from the compaction scale to the low-energy scale as a result of the effect of the light states in the theory. These are readily calculated. For example in the Calabi–Yau model (Greene *et al.* 1986) with a gauge breaking scale of not less than $O(10^{16}$ GeV) consistent with the constraint of baryon-number violation $\sin^2\theta_w = 0.225$, in good agreement with the experimental values. Reasonable values for $\sin^2\theta_w$ come too in various orbifold models and the flipped SU(5) model discussed above (Ibañez *et al.* 1987*a–c*, 1988; Bailin *et al.* 1987; Casas *et al.* 1987; Casas & Muñoz 1988; Antoniadis *et al.* 1988*b*).

6.4. *CP violation*

Dine & Seiberg (1986) have pointed out that strong CP violation is a generic problem for superstring models owing to small instanton contributions being enhanced by the growth of the strong coupling near the compactification scale. To avoid this, one may a try to use a Peccei–Quinn (PQ) symmetry allowing for the cancellation of the CP violation by the relaxation of the associated axion. However, Calabi–Yau compactification has no continuous (non-gauged) symmetries, so apparently there is no possibility of a PQ symmetry and hence of suppressing strong CP violation in such schemes (Choi & Kim 1985). However, the discrete symmetries of the model may give rise to an approximate PQ symmetry (broken only by non-renormalizable terms), which is quite adequate for suppressing CP violation (Lazarides *et al.* 1986; Casas & Ross 1987). Moreover, the natural size of PQ breaking is of $O(10^{11}$ GeV), which gives an easy explanation of the scale needed for a viable invisible axion. It remains to be seen whether this mechanism is realized in the three-generation models discussed here.

6.5. *Light Higgs scalars and electroweak breaking*

It is no surprise that three families of quarks and leptons remain light, because until electroweak breaking, the states in unpaired 27s (three families by construction) must remain massless. However, the Higgs doublets $H_1 = (h_1^+, h_1^0)$ and $H_2 = (h_2^0, h_2^-)$ needed for electroweak breaking; may couple in an $SU(3) \times SU(2) \times U(1)$ invariant way, giving H_1 and H_2 mass, and so it is surprising that any such components of a 27 remain light after large-scale breaking. The reason one pair escapes a large mass is that the symmetries of the model prevent an H_1–H_2 interaction until a high (non-renormalizable) order.

In the Calabi–Yau models the residual discrete symmetries guarantee this (Greene *et al.* 1986), whereas in the flipped SU(5) model, it is the missing partner mechanism (Masiero *et al.* 1982; Grinstein 1982; Kounnas *et al.* 1983) following from the gauge symmetries that keeps the Higgs light. In the orbifold models, too, it is the residual gauge symmetry that protects the light Higgs.

The triggering of electroweak breaking proceeds in the usual way via supersymmetry breaking from the hidden sector generating a negative squared mass, $-m^2$, for the Higgs scalars. This time, however, there is no D flat direction as there are no residual $\overline{27}$ components left light and so the potential $V(H)$ has the form $-m^2|H|^2 + g^2|H|^4$ leading to the expectation that the VEV of H should be of $O(m)$, the supersymmetry breaking mass scale. This scale is related to the scale at which a gaugino condensate occurs in the hidden sector, i.e. where the hidden-sector gauge coupling becomes large. This in turn is related to the compactification scale times the factor $\exp(-3S/b_0)$, where b_0 is the coefficient of the hidden sector β function and S is a field related to the dilation field and the field setting the scale of compactification, which in turn sets the scale for the gauge coupling. For large S/b_0 a large hierarchy of masses may arise and model calculations suggest (Ross 1988) it is possible to achieve the very small electroweak breaking scale required.

6.6. *Quark and lepton masses*

Perhaps the most exciting prospect for compactified string theories is that they should determine the quark and lepton mass matrices allowing for a precise test of the theory. In practice this is often difficult to implement for the techniques needed to determine the couplings have not been developed in every case. Within these limitations the mass spectra in the various models have been explored with encouraging results. As discussed in §5 the orbifold (Ibañez *et al.* 1987*a–c*, 1988; Bailin *et al.* 1987; Casas *et al.* 1987; Casas & Muñoz 1988), and fermionic models (Antoniadis *et al.* 1988*b*; Leontaris & Nanopoulos 1988; Leontaris 1988) may give rise to reasonable mass matrices, though no definite predictions have been determined. In the Calabi–Yau model, too, a reasonable mass spectrum emerges (Greene *et al.* 1986, 1987) and even a satisfactory prediction relating mixing angles results. However, the only model whose predictions are well established is the tensor product model of §5.4, so we will concentrate on this model to illustrate how the mass matrices are determined (Lutken & Ross 1989).

The first step is to determine the light particle spectrum after the stages of large spontaneous-gauge symmetry breaking. This consists of just three quark doublets, the extra D quarks getting large mass via the known trilinear couplings. In addition there is left light one pair of Higgs doublets H_1 and H_2 given by

$$\left.\begin{aligned} [H_1]_i &= [a_3\lambda_3 + a_5\lambda_5 + a_1\lambda_1]_{(i,1)}, \\ [H_2]_i &= [a_3'\lambda_3 + a_5'\lambda_5 + a_1'\lambda_1 + a_2'\lambda_2]_{(i,2)}, \end{aligned}\right\} \tag{6.4}$$

where λ_1, λ_3 and λ_5 are three of the original nine $(1,3,\bar{3})$ multiplets and are mixing angles determined by the Yukawa couplings and the large VEVs. The reason the primed and unprimed values may differ is due to the fact that with two stages of spontaneous symmetry breaking along $(\lambda)_{(1,3,3)}$ and $(\lambda)_{(1,3,2)}$ directions there may be masses $\lambda_{(i,1)}\lambda_{(i,2)}\langle\lambda_{(3,3)}\rangle$ and $\lambda_{(i,1)}\lambda_{(i,3)}$ $\langle\lambda_{(3,2)}\rangle$ destroying the symmetry between the components of (6.4). This has the important consequence that up and down quark mass matrices may differ.

Finally, there are left light three lepton doublets, mixtures of the nine $(1,3,\bar{3})$ multiplets and their neutrino singlet partners together with three heavy leptons.

Following from equation (6.4) and the determination of the Yukawa couplings, k_i, it is easy to write down the quark mass matrices. For the up quarks,

$$[Q_1\,Q_2\,Q_3]\begin{bmatrix} 0 & a_5 k_0 & 0 \\ 0 & 0 & a_1 k_0 \\ a_5 k_1 & a_1 k_0 & a_3 k_2 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}. \tag{6.5}$$

Here $k_0 = 0.6$, $k_1 = 0.5$, $k_2 = 1.05$ are the non-vanishing Yukawa couplings. If I fix the constants a_i by the three up quark masses we obtain predictions for the mixing angles of the (left-handed) up quarks. Doing the same for the down quarks gives finally a mixing matrix of the form.

$$\begin{bmatrix} 1 & \lambda & A\lambda^2\rho \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1-\rho) & -A\lambda^2 & 1 \end{bmatrix}, \tag{6.6}$$

where

$$\left.\begin{aligned} A\lambda^2 &= m_{\mathrm{d}}/m_{\mathrm{s}} - m_{\mathrm{u}}/m_{\mathrm{c}} = 0.04 \pm 0.002, \\ A\lambda\rho^2 &= m_{\mathrm{d}}/m_{\mathrm{b}} - m_{\mathrm{u}}/m_{\mathrm{t}} = 0.8 \pm 0.4, \end{aligned}\right\} \tag{6.7}$$

the numbers being experimental measurements. For $m_{\mathrm{b}} = 5$ Gev, $m_{\mathrm{s}} = 150$ MeV, $m_{\mathrm{d}} = 7.5$ MeV we get excellent agreement.

The magnitude of a_i, a_i' and hence the quark masses are related to the large VEVs breaking the gauge symmetry. These are not entirely determined by the calculated trilinear terms but depend also on the non-renormalizable terms $(27\cdot\overline{27})^n$. However, the VEVs have upper bounds corresponding to the scales at which the m^2 becomes negative. These scales are, in turn, determined by the trilinear couplings leading to the result, if we assume the upper bounds are saturated;

$$a_i/a_{j\alpha} \sim \exp\{-4\pi^2[1/\sum_\alpha c_\alpha(k_i^\alpha)^2_\beta - 1/\sum c_\beta(k_j^\beta)^2], \tag{6.8}$$

where k_i^α is the Yukawa coupling involved in the one-loop corrections with virtual states α driving the evolution of m^2 and c_α are constants associated with the Feynman graph. The quark masses are given in terms of the a_i: $m_{\mathrm{c}}/m_{\mathrm{t}} = a_5 k_0/a_3 k_2$, $m_{\mathrm{u}}/m_{\mathrm{c}} = a_1 a_k k_0^2/a_3 k_2$ and so it is easy to get large hierarchies of masses from the exponential factors in (6.8). Detailed predictions require a more careful treatment of the renormalization group equations than given in (6.8) and this has not yet been done but the expectation is that m_{t} will be of $O(M_{\mathrm{W}})$.

7. Summary and conclusions

The attempts to determine the low-energy model from the superstring are beset by the problems involved in determining the several stages of symmetry breaking, both of the underlying space-time, and of the supersymmetry and gauge symmetries. As a result of these problems, the candidate four-dimensional low-energy theories are very numerous. What is needed is an understanding of the mechanics, choosing a (unique?) vacuum state. In this, supersymmetry-breaking effects must play a crucial role when a selection is made among the many degenerate supersymmetric vacuum solutions.

Unfortunately, our understanding of supersymmetry breaking is limited, both because it requires a non-perturbative trigger and because the full form of the effective lagrangian, including string effects and heavy Kaluza–Klein exchange effects, is not known. However, some progress has been made; for example, it is known how supersymmetry-breaking effects can select a particular complex structure, thus choosing one of a class of Calabi–Yau compactifications.

In the absence of a complete understanding of the choice of vacuum state, it seems reasonable to use phenomenological criteria to select a promising scheme, and to investigate that scheme in detail as an example of the problems and answers that may be expected from the 'true' compactification.

The low-energy phenomenology of the candidate three-generation models analysed have certain features in common. All appear to have a large scale of symmetry breaking after compactification, leading to the possibilities of structure in the mass matrices. It also appears likely that this breaking will substantially reduce the low-energy spectrum; for example, it is no longer thought so likely that there will be additional U(1) gauge bosons at low energies, beyond the standard model $SU(3) \times SU(2) \times U(1)$ structure. Similarly, the presence of new exotic fermion states is no longer mandated by string compactification.

The detailed structure of the mass matrices in the tensor product model analysed has the correct form to explain the mixing angle and the hierarchy of fermion masses although it remains to be seen whether a complete analysis of the renormalization-group equations will give, in detail, good post-dictions for these quantities. At the very least the analysis shows how superstring models may realize their promise and give predictions going beyond the standard model.

It would be ridiculous, at this stage, to claim that any one model is *the* effective low-energy model following from the superstring, but its existence does show that the low-energy structure emerging from an underlying compactified theory can closely resemble the observed world. Moreover, it shows how the underlying structure may realize its promise and provide answers to the questions posed by the standard model, namely the origin of its multiplet structure and its parameters, the gauge and Yukawa couplings, the masses, and the mixing angles.

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Discussion

J. R. Ellis, F.R.S. (*Theory Division, CERN, Switzerland*). It seems to me that supersymmetry breaking is one of the most important open questions in superstring phenomenology. In particular, gaugino condensation alone seems to be inadequate, as it gives a potential which is unstable except when the gauge coupling vanishes. Does Dr Ross have any comment on this?

G. G. Ross. I agree that supersymmetry breaking is of great importance in determining the low-energy structure of the theory. Although gaugino condensation by itself is inadequate the inclusion of (string world-sheet) non-perturbative effects can give a potential with a stable minimum for non-vanishing gauge coupling. A model calculation suggests this has the structure needed to generate a large hierarchy of masses (Ross 1988*a*).

J. R. Ellis, F.R.S. I have predictions for m_b/m and for m_t. Does Dr Ross have any such predictions?

G. G. Ross. The predictions for quark masses may be obtained in the manner detailed in §6.7, but the detailed renormalization group analysis needed to determine these predictions has still to be performed.